

Meta-Pluralism: Moving Beyond the Universe/Multiverse Debate?

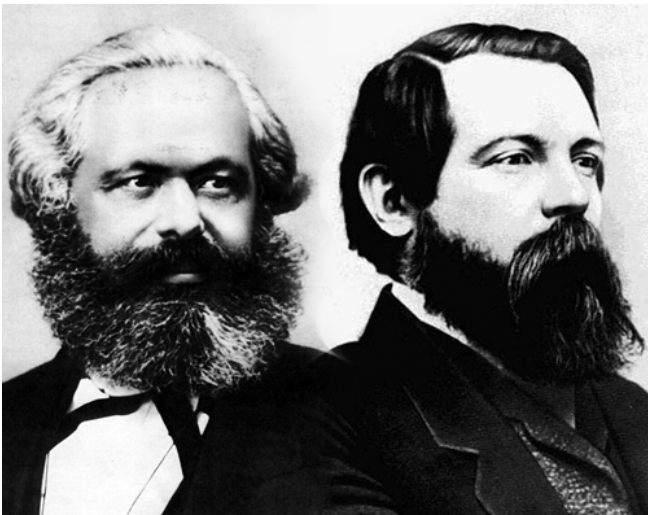
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J. D. Hamkins' Masterclass, Universität Konstanz

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The Spectre of the Multiverse

“A spectre is haunting maths – the spectre of the multiverse!”



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- It can be argued that supporters of either view seem to have *good* arguments to hold tight to it
- The goal of this paper is to challenge the *naturalness* and *inevitability* of this opposition
- The hope is to make some progress by *fine-tuning* and *explicating* basic ontological distinctions

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- There is a *single* ‘concept of set’
- There is a *single* collection of set-theoretic *truths*
- Every set-theoretic statement has a *determinate* truth-value (e.g., as a consequence of *categoricity results* about ZFC_2 , cf. the argument in [Zermelo, 1930] and [Martin, 2001])

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- There are many *concepts* of set
- For any theory T , there exists *multiverse*(T), the collection of all models of T
- Not *all* set-theoretic statements have a determinate truth-value (the axioms of set theory are *referentially indeterminate*, cf. the argument in [Putnam, 1980])

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- Universes are higher-order platonic entities
- The multiverse is a 'perspectival' phenomenon: *through* (and *inside*) each universe of set theory, one may explore the whole of the set-theoretic landscape (cf. [Hamkins, 2012], p. 417)

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- (Logic) Categoricity arguments are logically controversial (so referential indeterminacy doesn’t go away)
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- (Platonism) Plenitudinous versions of Platonism are fraught with problems
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- (Overgeneration) The multiverse entails an uncontrolled proliferation of entities
- (Redundancy) Whatever construction is carried out in the multiverse may also be carried out in V

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- U.-ist claims to have access to a *determinate* structure, V , but isn't able to pin down *fundamental* features of it.
- M.-ist claims to have access to several structures, but all of these *arise from* (or *are contained by*) V .

The Stalemate

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Is there any mistake we are committing, or logical issue we're not considering, which prevents further progress?

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- [Ryle, 1938] and [Ryle, 1949] are fundamental outlines of the concept (but cf. [Magidor, 2013], Ch. 1)
- Ryle identified ‘category mistakes’ as ways to produce *demarcations* among ontological domains (and, as a consequence, resolve key philosophical disputes)

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Broadly speaking (but cf. also Ryle's own interpretation), the mistake consists in an 'objectification' of the University.

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“Someone asks you to show them the universe of sets. You first show them the empty set, then a way to produce other sets from it, through a rule called ‘power-set’, so as to generate, among these new objects, some objects called ‘ordinals’. Now he takes note of the iteration of the process using the ordinals and, at certain ordinals (the limit-ordinals), he collects all the objects which have been created thus far, and so on. ‘Ok, good, I understand how sets are formed, but now tell me: where is the universe of sets?’.”

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Each of these constructions is routine in set theory. All of these show that set-theorists ordinarily deal with a V which has been ‘objectified’.

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Again, taken at face value, actualism about V seems to be equally unwarranted: V is *vastly indeterminate* (not in the sense referred to by the potentialist).

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Category Mistake (final version). V and the V -models belong to the *same* category.

V and V-models

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One misconception is: 'V is the actualist's (absolutist's) Absolute'.

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(Incidentally, this could be a vindication of [Kant, 1781]’s ‘transcendental argument’ for GU: if geometry is ‘transcendental geometry’, then Kant’s GU is right (the argument doesn’t go through if ‘transcendental geometry’ is replaced by ‘Euclidean geometry’’).

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Comment: By the ontological picture above, this statement is vague, so one is entitled to reject it.

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Comment: Sure, there is no single *determinate* set-theoretic truth ('truth in V ' is indeterminate).

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A potential response in the next slide.

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- Do categoricity arguments really entail *determinacy*?

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Again, do these results indicate that categoricity entails determinacy?

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- V isn't a determinate *object* (not even a determinate *structure*)
- Categorical theories do not afford *determinacy*, but *transcendence* of V
- The way second-order set theory captures the notion of V only does justice to the *transcendence* of V

Determinacy vs. Transcendence

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Comment: if we can't know the objects in V , only 'structural properties', don't we also know V in a fully indeterminate way?

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As we have seen, we need a richer (more articulated) account of set-theoretic ontology.

So far, we have identified three different, *co-existing*, ontological constructs: 1) V , the: 2) V -models, and: 3) further models (jointly taken to constitute the ‘multiverse’).

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- 1) *Monism*: there is one set-theoretic universe.
- 2) *Pluralism*: there are many set-theoretic universes.
- 3) *Meta-pluralism*: there are many set-theoretic ontological constructs, with different degrees of *determinacy* (and *transcendancy*).

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This goal is marvellously expressed by the ‘multiverse undertaking’.

The investigation (and classification) of different roles for different concepts, in a ‘meta-pluralistic’ fashion, just adds to the extraordinary *foundational richness* of set theory.

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- this leads potentially to view the U.-M. *opposition* as flawed
- a more articulated picture of set-theoretic ontology based on the distinction above is needed

End Slide

Thanks!

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